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Aerodynamic models for Darrieus-type straight-bladed vertical axis wind turbines

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Abstract

Since ancient past humans have attempted to harness the wind energy through diversified means and vertical axis wind turbines (VAWTs) were one of the major equipment to achieve that. In this modern time, there is resurgence of interests regarding VAWTs as numerous universities and research institutions have carried out extensive research activities and developed numerous designs based on several aerodynamic computational models. These models are crucial for deducing optimum design parameters and also for predicting the performance before fabricating the VAWT. In this review, the authors have attempted to compile the main aerodynamic models that have been used for performance prediction and design of straight-bladed Darrieus-type VAWT. It has been found out that at present the most widely used models are the double-multiple streamtube model, Vortex model and the Cascade model. Each of these three models has its strengths and weaknesses which are discussed in this paper.

Keywords: Renewable energy; Vertical axis wind turbine; VAWT; Wind; Straight-bladed; Darrieus

Contents

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1. Introduction

At present, there are two categories of modern wind turbines, namely horizontal axis wind turbines (HAWTs) and vertical axis wind turbines (VAWTs), which are used mainly for electricity generation and pumping water. The main advantage of VAWT is its single moving part (the rotor) where no yaw mechanisms are required, thus simplifying the design configurations significantly. Blades of straight-bladed VAWT may be of uniform section and untwisted, making them relatively easy to fabricate or extrude, unlike the blades of HAWT, which should be twisted and tapered for optimum performance. Furthermore, almost all of the components of VAWT requiring maintenance are located at the ground level, facilitating the maintenance work appreciably.

From the past experiences, it is evident that wind turbines can compete with conventional sources in niche markets, and lower costs make them affordable options in increasingly large markets. Environmentally benign VAWTs can be utilized for a range of applications, including (i) electricity generation; (ii) pumping water; (iii) purifying and/or desalinating water by reverse osmosis; (iv) heating and cooling using vapour compression heat pumps; (v) mixing and aerating water bodies; and (vi) heating water by fluid turbulence. In general, VAWT can sensibly be used in any area with sufficient wind, either as a stand-alone system to supply individual households with electricity and heat, or for the operation of freestanding technical installations. If a network connection is available, the energy can be fed in, thereby contributing to a reduction in electricity costs. In order to maximize the security of the energy supply, different types of VAWT can be supplemented by a photovoltaic system or a diesel generator in a quick and uncomplicated fashion. Through the combination of several VAWTs with other renewable energy sources and a backup system, local electrical networks can be created for the energy supply of small settlements and remote locations.

Abbreviations and Acronyms

A	projected frontal area of turbine
AR	aspect ratio $= H/C$
C	blade chord
C_d	blade drag coefficient
C_{dor}	reference zero-lift-drag coefficient
C_D	turbine overall drag coefficient $= F_D/\rho A V_\infty^2$
C_{DD}	rotor drag coefficient $= F_D/\rho A V_\infty^2$
C_l	blade lift coefficient
C_n	normal force coefficient
C_P	turbine overall power coefficient $= P_o/\rho A V_\infty^3$
C_Q	turbine overall torque coefficient $= Q/\rho A V_\infty^2 R$
C_t	tangential force coefficient
d	minimum distance from the vortex filament
D	blade drag force
$\vec{e}$	unit vector
F	force on blade airfoil
F_D	turbine overall drag force
F_n	normal force (in radial direction)
F_t	tangential force
F_{ta}	average tangential force
F_t	non-dimensional tangential force $= C_t\ (W/V_\infty)^2$
H	height of turbine
HAWT	horizontal axis wind turbine
k_i	exponent in the induced velocity relation
L	blade lift force
$\dot{m}$	mass flow rate
N	number of blade
p	static pressure
P_o	overall power
P_∞	atmospheric pressure
Q	overall torque
$\vec{r}$	unit vector
R	turbine radius
Re	local Reynolds number $= WC/\upsilon$
t	blade spacing $= (2\pi R/N)$
V	centre line velocity along freestream velocity direction
V_a	induced velocity
V_{ad}	induced velocity in the downstream side
V_{au}	induced velocity in the upstream side
V_c	chordal velocity component
V_{cd}	chordal velocity component in the downstream side
V_{cu}	chordal velocity component in the upstream side
V_e	wake velocity in upstream side
V_n	normal velocity component
V_{nd}	normal velocity component in the downstream side

V_{nu}	normal velocity component in the upstream side
$\vec{V}_p$	induced velocity at a point P on the filament
V_w	wake velocity in downstream side
V_Γ	velocity contributed by circulation
V_∞	wind velocity
VAWT	vertical axis wind turbine
W	relative flow velocity
W_d	relative flow velocity in the downstream side
W_u	relative flow velocity in the upstream side
α	blade angle of attack
α_d	blade angle of attack in the downstream side
α_u	blade angle of attack in the upstream side
γ	blade pitch angle
Γ	circulation per unit length
θ	azimuth angle
λ	tip speed ratio $= R\omega/V_\infty$
v	kinematic viscosity
ρ	fluid density
σ	solidity $= NC/R$
ω	angular velocity of turbine in rad/s

In this modern time, there is resurgence of interests regarding VAWTs as several universities and research institutions have carried out extensive research activities and developed numerous designs based on several aerodynamic computational models. These models are crucial for optimum design parameters and also for predicting the performance before fabricating the models or prototypes. In this paper, the authors attempt to explore the main aerodynamic models that have been used for performance and design analysis of straight-bladed Darrieus-type VAWT through literature survey that are organized and briefly described in the subsequent headings.

2. Historical background

Wind energy systems have been used for centuries as a source of energy for mankind. According to historic sources, the Babylonian emperor Hammurabi used windmills for an ambitious irrigation project as early as the 17th century BCE [1]. Later on, Persian inventors developed a wind-power machine, a more advanced windmill than that developed by the Babylonians [2]. Arab geographers traveling in Afghanistan in the 7th century wrote descriptions of windmills, which resembled our modern revolving doors [3]. Vertical windmills of this category were still used in some parts of Iran and Afghanistan in the late 1980s, and it was estimated that they generated about 75 hp and can grind a ton of wheat every 24 h [4].

The earliest European windmills appeared in France and England in the 12th century and quickly spread throughout Europe. These early wood structures, called post mills, were rotated by hand around a central post to bring the sails into the wind. By the late part of the 13th century the typical 'European windmill' had been developed and this became the norm until further developments were introduced during the 18th century. At the end

of the 19th century there were more than 30,000 windmills in Europe, used primarily for the milling of grain and water pumping [1].

3. Modern VAWT types

There have been many designs of vertical axis windmills over the centuries and currently the vertical axis wind turbines can be broadly divided into three basic types, namely (1) Savonius type, (2) Darrieus type, and (3) H-Rotor type. Brief descriptions of these VAWT types are given below.

3.1. Savonius wind turbine

The Savonius-type VAWT, as shown in Fig. 1, was invented by a Finnish engineer S.J. Savonius in 1929 [5]. It is basically a drag force driven wind turbine with two cups or half drums fixed to a central shaft in opposing directions. Each cup/drum catches the wind and so turns the shaft, bringing the opposing cup/drum into the flow of the wind. This cup/drum then repeats the process, causing the shaft to rotate further, thus completing a full rotation. This process continues all the time the wind blows and the turning of the shaft is used to drive a pump or a small generator. Though typical values of maximum power coefficient for other types of wind turbines vary between 30% to 45%, those for the Savonius turbines are typically not more than 25% according to most investigators [6]. This type of turbine is suitable for low-power applications and they are commonly used for wind speed instruments.

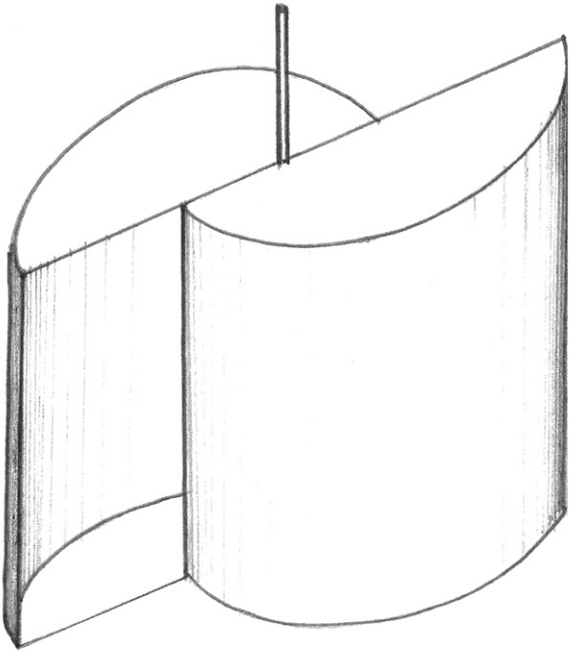


Fig. 1. Savonius-type VAWT.

3.2. Darrieus wind turbines

The modern Darrieus VAWT was invented by a French engineer George Jeans Mary Darrieus. He submitted his patent in 1931 [7] in the USA which included both the "Eggbeater (or Curved Bladed)" and "Straight-bladed" VAWTs. Sketches of these two variations of Darrieus concepts are shown in Figs. 2 and 3, respectively. The Darrieus-type VAWTs are basically lift force driven wind turbines. The turbine consists of two or more aerofoil-shaped blades which are attached to a rotating vertical shaft. The wind blowing over the aerofoil contours of the blade creates aerodynamic lift and actually pulls the blades along. The troposkien shape eggbeater-type Darrieus VAWT, which minimizes the bending stress in the blades, were commercially deployed in California in the past.

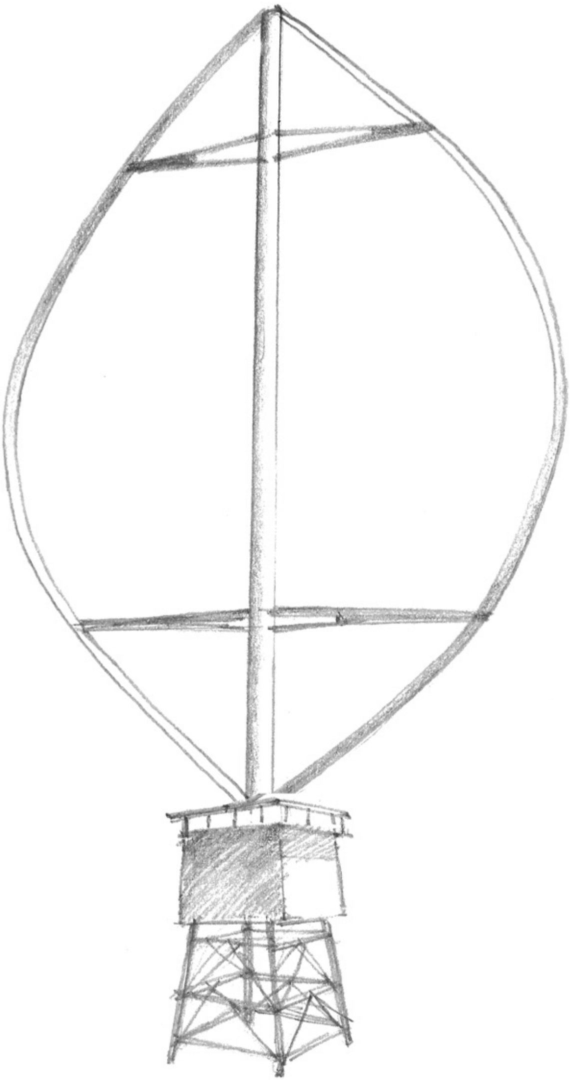


Fig. 2. Curved-blade (or "Egg-beater" type) Darrieus VAWT.

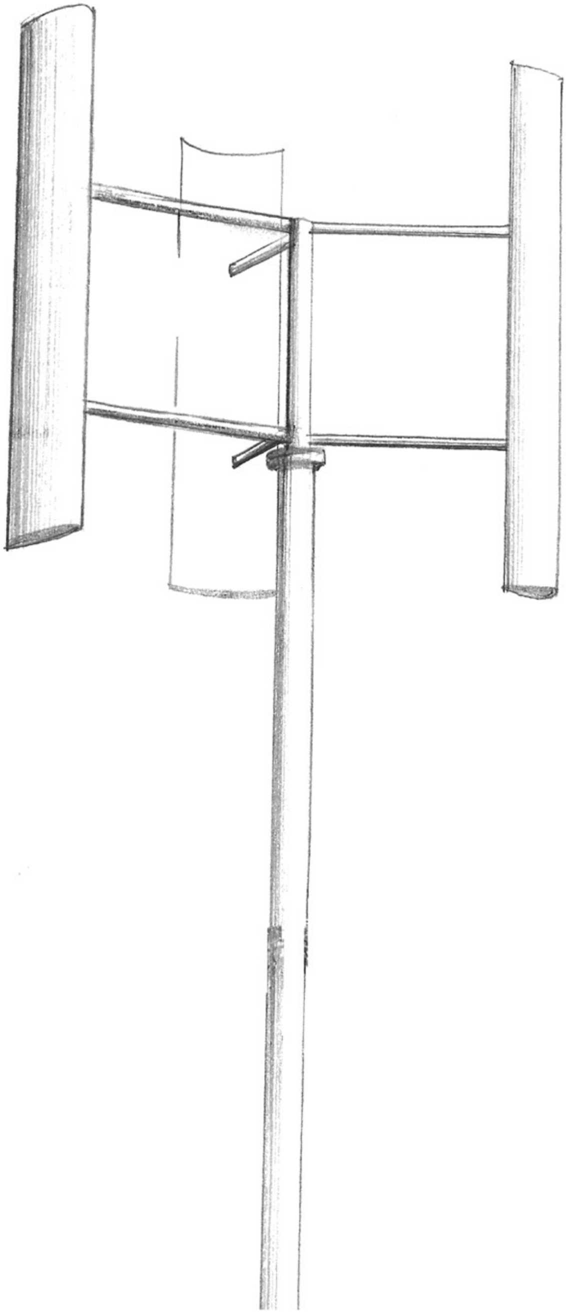


Fig. 3. Straight-bladed Darrieus VAWT.

In the small-scale wind turbine market, the simple straight-bladed Darrieus VAWT, often called giromill or cyclo-turbine, is more attractive for its simple blade design. This configuration fall into two categories: fixed pitch and variable pitch. It has been found out

from the previous research activities that fixed pitch VAWTs provide inadequate starting torque [6]. Contemporary variable pitch blade configuration has potential to overcome the starting torque problem but it is overly complicated, rendering it impractical for smaller capacity applications. Majority of the previously conducted research activities on VAWT focused on straight bladed VAWTs equipped with symmetric airfoils (like NACA 4-digit series of 0012, 0015, 0018) which were unable to self-start. This inability to self-start is due to several factors (like technical, inadequate research work & funding), and the most dominant ones are due to aerodynamic factors. According to Internet survey, there are handfuls of commercial straight-bladed VAWTs products, but no reliable information could be obtained from an independent source regarding the performance of these products and the claims made by the manufacturers are yet to be authentically verified.

At present, development of large-scale straight-bladed VAWT is limited to the research stage only, although large eggbeater Darrieus wind turbine had reached the market commercially in the past before disappearing away later. However, in the small-scale wind turbine market, the simple straight-bladed Darrieus seems to be more cost effective than the eggbeater Darrieus as few companies had marketed this type of wind turbine before, i.e. the Pinson/Asi cycloturbine which utilized an end tail for changing pitch. This particular giromill model was stated in Drees' [8] research paper of having 3.6 m diameter and 2.4 m height. With 3 blades at chord length of 29 cm, the rotor has solidity of 0.24. Another pitch changing research prototype was built by Grylls et al. [9]. It has a diameter of 2.4 m and a height of 1.6 m. Using 3 blades with a chord length of 14.5 cm only, the rotor has a solidity of 0.18. Wind tunnel results for this prototype indicated the rotor was able to self-start at wind speed of 3.5 m/s, provided the pitch angle change is larger than plus or minus 4°.

3.3. H-Rotors

H-Rotors, as shown in Fig. 4, were developed in the UK through the research carried out during the 1970–1980s when it was established that the elaborated mechanisms used to feather the straight-bladed Darrieus VAWT blades were unnecessary. It was found out that the drag/stall effect created by a blade leaving the wind flow would limit the speed that the opposing blade (in the wind flow) could propel the whole blade configuration forward. The H-Rotor was therefore self-regulating in all wind speeds reaching its optimal rotational speed shortly after its cut-in wind speed.

4. General mathematical expressions for aerodynamic analysis of straight-bladed Darrieus-type VAWTs

Though the straight-bladed Darrieus-type VAWT is the simplest type of wind turbine, its aerodynamic analysis is quite complex. Before comparative analysis of the main aerodynamic models, the general mathematical expressions, which are common to most of the aerodynamic models, are described in this section.

4.1. Variation of local angle of attack

The flow velocities in the upstream and downstream sides of the Darrieus-type VAWTs are not constant as seen in Fig. 5. From this figure one can observe that the flow is

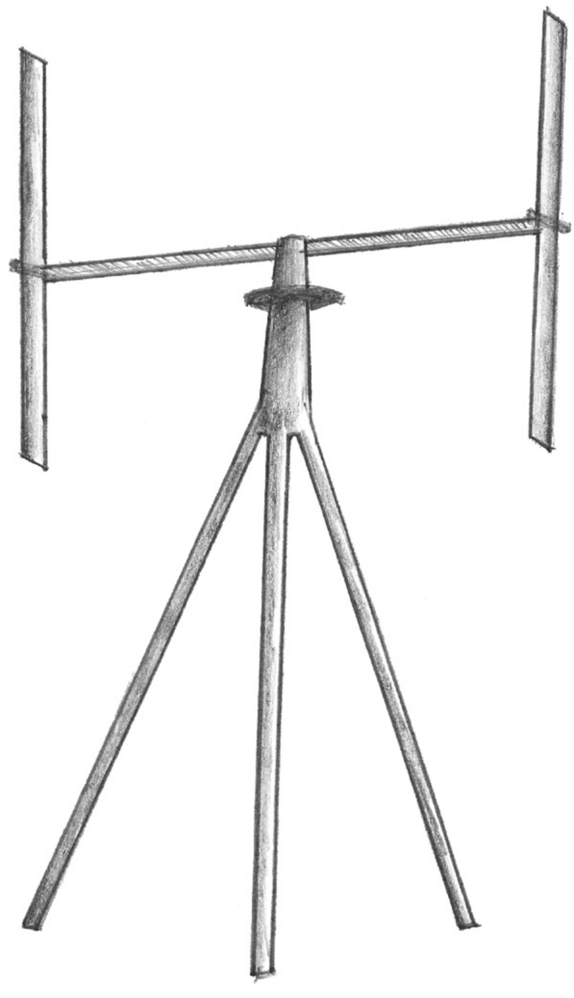


Fig. 4. H-Rotor-type VAWT.

considered to occur in the axial direction. The chordal velocity component V_c and the normal velocity component V_n are, respectively, obtained from the following expressions:

$$V_c = R\omega + V_a \cos\theta, \tag{1}$$

$$V_n = V_a \sin\theta, \tag{2}$$

where V_a is the axial flow velocity (i.e. induced velocity) through the rotor, ω is the rotational velocity, R is the radius of the turbine, and θ is the azimuth angle. Referring to Fig. 5, the angle of attack (α) can be expressed as

$$\alpha = \tan^{-1}\left(\frac{V_n}{V_c}\right). \tag{3}$$

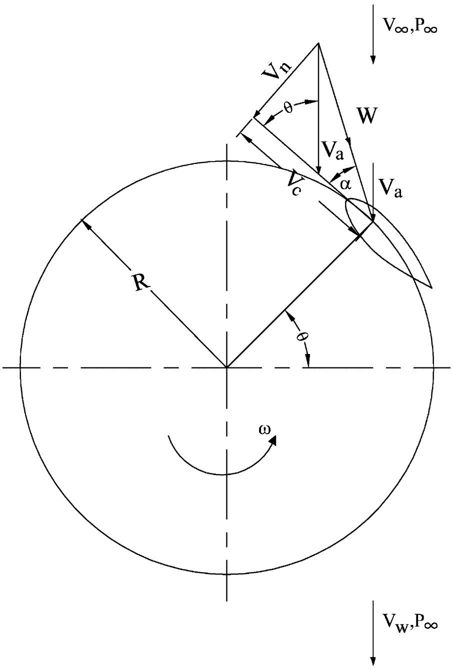


Fig. 5. Flow velocities of straight-bladed Darrieus-type VAWT.

Substituting the values of V_n and V_c and non-dimensionalizing,

$$\alpha = \tan^{-1}\left[\frac{\sin\theta}{(R\omega/V_\infty)/(V_a/V_\infty)+\cos\theta}\right], \tag{4}$$

where V_∞ is the freestream wind velocity. If we consider blade pitching then,

$$\alpha = \tan^{-1}\left[\frac{\sin\theta}{(R\omega/V_\infty)/(V_a/V_\infty)+\cos\theta}\right]-\gamma, \tag{5}$$

where γ is the blade pitch angle.

4.2. Variation of local relative flow velocity

The relative flow velocity (W) can be obtained as (Fig. 5),

$$W = \sqrt{V_c^2 + V_n^2}. \tag{6}$$

Inserting the values of V_c and V_n (Eqs. (1) and (2)) in Eq. (6), and non-dimensionalizing, one can find velocity ratio as,

$$\frac{W}{V_\infty} = \frac{W}{V_a}\cdot\frac{V_a}{V_\infty} = \frac{V_a}{V_\infty}\sqrt{\left[\left(\frac{R\omega}{V_\infty}\bigg/\frac{V_{au}}{V_\infty}\right)+\cos\theta\right]^2+\sin^2\theta}. \tag{7}$$

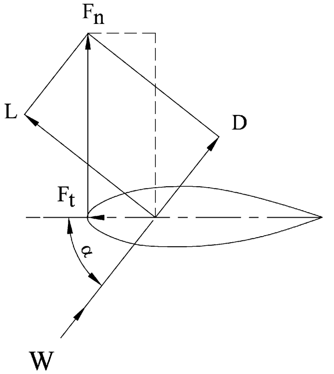


Fig. 6. Force diagram of a blade airfoil.

4.3. Variation of tangential and normal forces

The directions of the lift and drag forces and their normal and tangential components are shown in Fig. 6. The tangential force coefficient (C_t) is basically the difference between the tangential components of lift and drag forces. Similarly, the normal force coefficient (C_n) is the difference between the normal components of lift and drag forces. The expressions of C_t and C_n can be written as

$$C_t = C_l \sin\alpha - C_d \cos\alpha, \tag{8}$$

$$C_n = C_l \cos\alpha + C_d \sin\alpha. \tag{9}$$

The net tangential and normal forces can be defined as

$$F_t = C_t \tfrac{1}{2}\rho \mathrm{CHW}^2, \tag{10}$$

$$F_n = C_n \tfrac{1}{2}\rho \mathrm{CHW}^2, \tag{11}$$

where ρ is the air density, C is the blade chord and H is the height of the turbine.

4.4. Calculation of total torque

Since, the tangential and normal forces represented by Eqs. (10) and (11) are for any azimuthal position, so, they are considered as a function of azimuth angle θ. Average tangential force (F_{ta}) on one blade can be expressed as

$$F_{ta} = \frac{1}{2\pi}\int_0^{2\pi} F_t(\theta)\,\mathrm{d}\theta. \tag{12}$$

The total torque (Q) for the number of blades (N) is obtained as

$$Q = NF_{ta}R. \tag{13}$$

4.5. Power output

The total power (P) can be obtained as

$$P = Q \cdot \omega. \tag{14}$$

5. Computational models for Darrieus-type straight-bladed VAWT

In the past, several mathematical models, based on several theories, were prescribed for the performance prediction and design of Darrieus-type VAWTs by different researchers. The key components of all the computational models can be broadly described as:

- calculations of local relative velocities and angle of attacks at different tip speed ratios and azimuthal (orbital) positions;
- calculation of ratio of induced to freestream velocity (V_a/V_∞) considering the blade/blade-wake interaction;
- mathematical expressions based on different approaches (Momentum, Vortex or Cascade principles) to calculate normal and tangential forces;
- 'Pre-Stall airfoil characteristics' (C_l, C_d & C_m) for the attached regime at different Reynolds numbers;
- 'Post-Stall Model' for Stall Development and Fully Stalled regimes;
- 'Finite Aspect Ratio consideration';
- 'Dynamic Stall Model' to account for the unsteady effects;
- 'Flow Curvature Model' to consider the circular blade motion.

According to literature survey, the most studied and best validated models can be broadly classified into three categories—(1) Momentum model, (2) Vortex model and (3) Cascade model. It should be noted that not all the models consider all the key components described above. Descriptions of these three main categories of VAWT computational models are presented below.

5.1. Momentum model

Different momentum models (also specified as Blade Element/Momentum or BEM model) are basically based on calculation of flow velocity through turbine by equating the streamwise aerodynamic force on the blades with the rate of change of momentum of air, which is equal to the overall change in velocity times the mass flow rate. The force is also equal to the average pressure difference across the rotor. Bernoulli's equation is applied in each streamtube. The main drawback of these models is that they become invalid for large tip speed ratios and also for high rotor solidities because the momentum equations in these particular cases are inadequate [10]. Over the years, several approaches were attempted to utilize this concept, which are discussed briefly in the following headings.

5.1.1. Single streamtube model

In 1974 Templin proposed the single streamtube model which is the first and most simple prediction method for the calculation of the performance characteristics of a Darrieus-type

VAWTs [11]. In this model the entire turbine is assumed to be enclosed within a single streamtube as shown in Fig. 7. This model first incorporated the concept of the windmill actuator disc theory into the analytical prediction model of a Darrieus-type VAWT. In this theory the induced velocity (rotor axial flow velocity) is assumed to be constant throughout the disc and is obtained by equating the streamwise drag with the change in axial momentum.

In the assumption, the actuator disc is considered as the surface of the imaginary body of revolution. It is assumed that the flow velocity is constant throughout the upstream and downstream side of the swept volume. This theory takes into account the effect of airfoil stalling on the performance characteristics. The effects of geometric variables such as blade solidity and rotor height–diameter ratio have been included in the analysis. The effect of zero-lift-drag coefficient on the performance characteristics has also been included. Wind shear effect cannot be incorporated into the model.

Now, according to Gluert Actuator Disk theory, the expression of the uniform velocity through the rotor is

$$V_{\mathrm{a}} = \frac{V_{\infty} + V_{\mathrm{w}}}{2}, \tag{15}$$

where V_{w} is the wake velocity. All the calculations in this model are performed for a single blade whose chord equals the sum of the chords of the actual rotor's blades. The

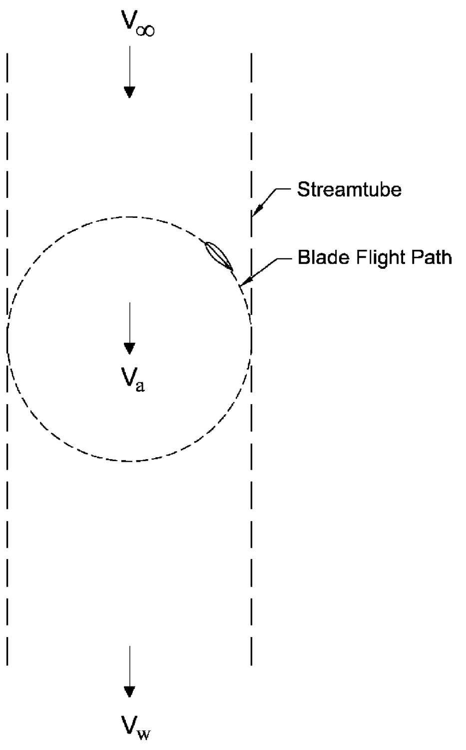


Fig. 7. Schematic of single streamtube model.

streamwise drag force (F_D) due to the rate of change of momentum is

$$F_D = \dot{m} \cdot (V_\infty - V_w), \tag{16}$$

where $\dot{m}$ ($= A\rho V_a$) is the mass flow rate. The rotor drag coefficient (C_{DD}) is defined as

$$C_{DD} = \frac{F_D}{1/2\rho A V_a^2}. \tag{17}$$

From Eqs. (16) and (17), we can find that

$$C_{DD} = 4\left(\frac{V_\infty - V_a}{V_a}\right) \tag{18}$$

and

$$\frac{V_a}{V_\infty} = \left(\frac{1}{1 + C_{DD}/4}\right). \tag{19}$$

The overall torque and power coefficient of the VAWT can be determined from Eqs. (13) and (14) by utilizing the expression of V_a/V_∞ derived in Eq. (19) above.

This model can predict the overall performance of a lightly loaded wind turbine but according to the inquest, it always predicts higher power than the experimental results. It does not predicts the wind velocity variations across the rotor. These variations gradually increase with the increase of the blade solidity and tip speed ratio. In 1980, Noll and Ham presented an analytical method for the performance prediction of a cyclically pitched straight-bladed vertical-axis wind turbine using the single streamtube model [12]. They added the effect of strut drag, turbulent wake state and dynamic stall to their analytical method.

5.1.2. Multiple streamtube model

In 1974, Wilson and Lissaman [13] introduced the Multiple streamtube model which was an improvement to single streamtube model. In this model the swept volume of the turbine is divided into a series of adjacent, aerodynamically independent parallel streamtubes as shown in Fig. 8. The blade element and momentum theories are then employed for each streamtube. In their model they considered the flow as inviscid and incompressible for the calculation of the induced velocity through the streamtube. As a result, there appears only the lift force in the calculation of the induced velocity. Wilson and Lissaman [13] considered the theoretical lift for their calculation, which is given by

$$C_l = 2\pi \sin \alpha. \tag{20}$$

In this model, the induced velocity ratio can be found from the following expression:

$$\frac{V_a}{V_\infty} = 1 - \left(\frac{k}{2} . \frac{Nc}{R} . \frac{R\varpi}{V_\infty} . \sin \theta\right), \tag{21}$$

where k is a factor found through iteration. In this model, the induced velocity varies over the frontal disc area both in the vertical and horizontal directions [14]. Atmospheric wind shear can be included in this model. However, this model still is inadequate in its description of flow field and it can be applied only for a fast running lightly loaded wind turbine.

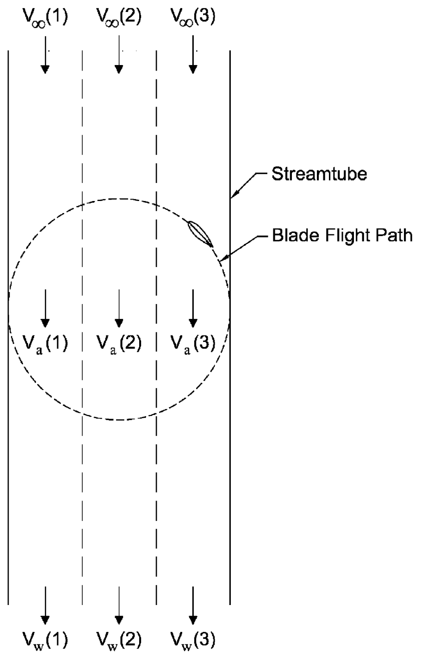


Fig. 8. Schematic of multiple streamtube model.

In 1975, Strickland [15] presented another multiple streamtube model for a Darrieus-type VAWT. In this model, induced velocity is found by equating the blade elemental forces (including airfoil drag) and the change in the momentum along each streamtube. The wind shear effects have also been included in the calculation of the model. This model predicts the overall performance reasonably, especially when the rotor is lightly loaded. It displays improvement over the single streamtube model.

The basic difference between Wilson's and Strickland's models is that Wilson used the theoretical lift force only in the calculation of induced velocity while Strickland added the effect of drag force as well for the similar calculation. Among these two models, Wilson's model gives fast convergence while Strickland's model gives slow convergence due to added complicacy.

Another theory based on the multiple streamtube model including the effects of airfoil geometry, support struts, blade aspect ratio, turbine solidity and blade interference was presented by Muraca et al. [16]. The effect of flow curvature is evaluated by considering the flow over a flat plate. They derived an expression of lift distribution on the plate with the variable angle of attack from the leading to the trailing edge points of the flat plate and averaged the distributed lift force over the whole surface of the flat plate. According to them, the effect of flow curvature on the performance characteristics is insignificant for a low chord to radius ratio.

In 1977, Sharpe gave an elaborated description of the multiple streamtube model in a report. The principal idea of his model is similar to Strickland's model. Additionally, he incorporated the effect of Reynolds number into the calculation [17]. In 1980, another improved version of the multiple streamtube model was presented by Read and Sharpe [18]

for VAWT. In their model the parallel streamtube concept is dispensed with and the expansion of the streamtube is included. It is strictly applicable to low solidity lightly loaded wind turbines with large aspect ratio (H/c). It can predict the instantaneous aerodynamic blade forces and the induced velocities better than those by the conventional multiple streamtube model. But the prediction of overall power coefficients cannot be made with reasonable accuracy. It usually gives lower power than that obtained experimentally.

5.1.3. Double-multiple streamtube model

In 1981, Paraschivoiu [19] introduced double multiple streamtube theory for the performance prediction of a Darrieus wind turbine. As shown in Fig. 9, in this model the calculation is done separately for the upstream and downstream half cycles of the turbine. At each level of the rotor, the upstream- and downstream-induced velocities are obtained using the principle of two actuator discs in tandem. The concept of the two actuator discs in tandem for a Darrieus wind turbine was originally given by Lapin [20]. For both the upstream and downstream half cycles vertical variation of the induced velocity (like that in the multiple stream tube model) is considered while in the horizontal direction induced velocity is assumed to be constant (like that of a single streamtube model). For the upstream half-cycle, the wake velocity is represented by

$$V_e = V_{\infty i}\left(2\frac{V_{au}}{V_{\infty i}} - 1\right) = V_{\infty i}(2u_u - 1), \tag{22}$$

where $V_{\infty i}$ is the local ambient wind velocity (which is different at different heights of the turbine bladed due to the effect of wind shear), V_{au} is the induced velocity and $u_u(= V_{au}/V_{\infty i})$ is the interference factor for the upstream half-cycle. For the downstream half-cycle of the rotor, V_e is the input velocity. The induced velocity for the downstream half-cycle is V_{ad} which can be written as

$$V_{ad} = u_d V_e = u_d(2u_u - 1)V_{\infty i}, \tag{23}$$

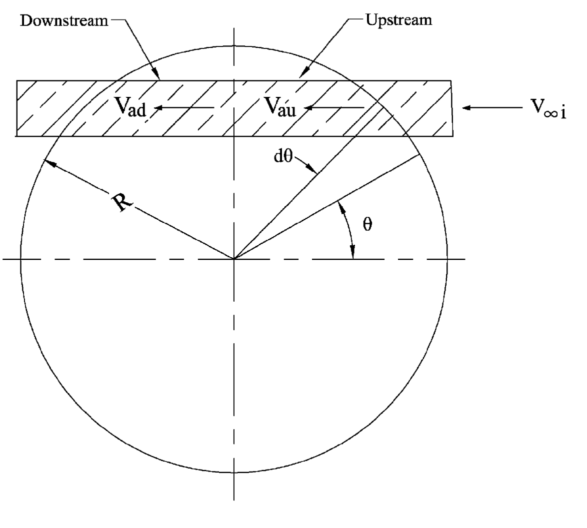


Fig. 9. Schematic of double-multiple streamtube model.

where, $u_d(=V_{ad}/V_e)$ is the interference factor for the downstream half-cycle. The streamtube induced velocity is calculated by a double iteration, one for each part of the rotor.

The double-multiple streamtube model with constant and variable interference factors (induced velocity ratios), including secondary effects for a Darrieus wind rotor was examined by Paraschivoiu et al. [21]. They found a relatively significant influence of the secondary effects, namely, the blade geometry and profile type, the rotating tower and the presence of struts and aerodynamically spoilers, especially at high tip speed ratios. They considered the variation of the induced velocity as a function of azimuth angle that gives a more accurate calculation of the aerodynamic loads. In the paper presented by Paraschivoiu and Delclaux [22], they made improvements in the double-multiple streamtube model. They considered the induced velocity variation as a function of the azimuth angle for each streamtube.

The double-multiple streamtube model gives better correlation between the calculated and experimental results, especially for the local aerodynamic blade forces with the multiple streamtube models. However, this model gives over prediction of power for a high solidity turbine and there appears to be a convergence problem for the same type of turbine especially in the downstream side and at the higher tip speed ratio.

5.2. Vortex model

The Vortex models are basically potential flow models based on the calculation of the velocity field about the turbine through the influence of vorticity in the wake of the blades. The turbine blades are represented by bound or lifting-line vortices whose strengths are determined using airfoil coefficient datasets and calculated relative flow velocity and angle of attack.

A simple representation of the vortex system associated with a blade element is shown in Fig. 10. The VAWT blade element is replace by a "bound" vortex filament sometimes called "substitution" vortex filament or a lifting line. The strengths of the bound vortex and each trailing tip vortex are equal as a consequence of the Helmholtz theorems of voticity [23]. According to Fig. 10, the strengths of the shed vortex systems have changed on several occasions. On each of these occasions, a spanwise vortex is shed whose

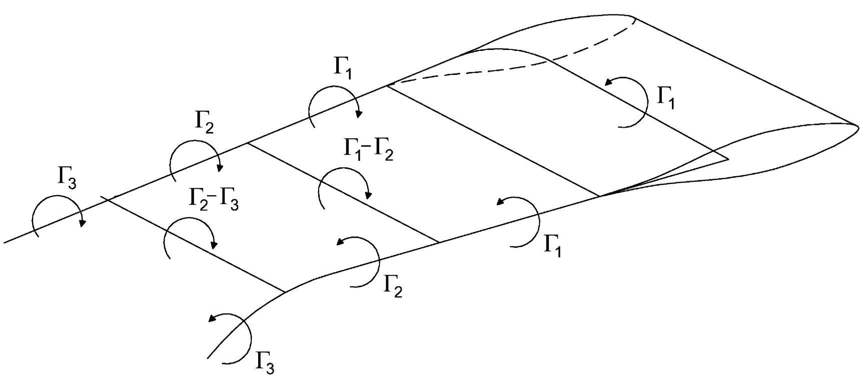


Fig. 10. Vortex system for a single blade element.

strength is equal to the change in the bound vortex strength as dictated by Kelvin's theorem [23].

The fluid velocity at any point in the flow field is the sum of the undisturbed wind stream velocity and the velocity induced by the entire vortex filaments in the flow filed. The velocity induced at a point in the flow field by a single vortex filament can be obtained from the Biot–Savart law, which relates the induced velocity to the filament strength. Referring to the case shown in Fig. 11, for a straight vortex filament of strength Γ and length l, induced velocity $\vec{V}_{\mathrm{p}}$ at a point P on the filament is given by,

$$\vec{V}_{\mathrm{p}} = \vec{e}\frac{\Gamma}{4\pi d}(\cos\theta_1 - \cos\theta_2), \tag{24}$$

where d is the minimum distance of the point P from the vortex filament, $\vec{e}$ and $\vec{r}$ are the unit vectors. It should be noted that if point P should happen to lie on the vortex filament, Eq. (24) yields indeterminate results, since $\vec{e}$ cannot be defined and the magnitude of $\vec{V}_{\mathrm{p}}$ is infinite. The velocity induced by a straight filament on itself is, in fact, equal to zero.

In order to allow closure of the Vortex model, a relationship between the bound vortex strength and the velocity induced at a blade segment must be obtained. A relationship between the lift L per unit span on a blade segment and the bound vortex strength Γ_{B} is given by the Kutta–Joukowski law. The lift can also be formulated in terms of the airfoil lift coefficient C_{L}. Equating these two expressions for the lift, yields the required relationship between the bound vortex strength and the induced velocity at a particular blade segment as,

$$\Gamma_{\mathrm{B}} = \tfrac{1}{2}cC_{\mathrm{L}}W, \tag{25}$$

where, c is the blade chord. After determining the induced velocity distribution, it becomes straight-forward to obtain performance characteristics of VAWT as described in Section 4.

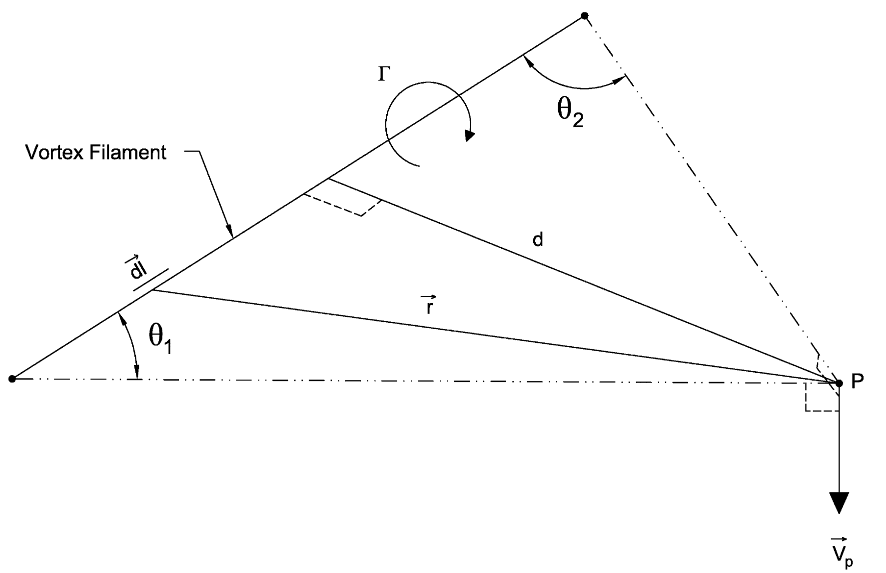


Fig. 11. Velocity induced at a point by a vortex filament.

In 1975, Larsen [24] first introduced the idea of Vortex model. The Vortex system for a single blade element of a VAWT. He used his Vortex model for the performance prediction of a cyclogiro windmill. His model is a two-dimensional one but if the vortex trailing from the rotor blade tips is considered, it may be said that it is not strictly two-dimensional. However, in his model angle of attack is assumed small, as a result, the stall effect is neglected.

Fanucci and Walters [25] presented a two-dimensional Vortex model applicable to a straight-bladed VAWT. In their analysis they considered the angle of attack very small which eliminates the stall effect. Holme [26] presented a Vortex model for a fast running vertical-axis wind turbine having a large number of straight, very narrow blades and a high height–diameter ratio (in order to make a two-dimensional flow assumption). The analysis is valid for a lightly loaded wind turbine only. Wilson [27] also introduced a two-dimensional vortex analysis to predict the performance of a giromill. In his method he did not take the stall effect into account, because the angle of attack was assumed to be small.

In 1979, Strickland et al. [28] presented an extension of the Vortex model which is a three-dimensional one and the aerodynamic stall is incorporated into the model. They presented the experimental results for a series of two-dimensional rotor configurations. Their calculated values show more or less good correlation with the experimental results for the instantaneous blade forces and the near wake flow behind the rotor. Strickland et al. [29] made improvements on the prior Vortex model (quasi-steady Vortex model). The latest model is termed as the dynamic Vortex model, since, in this model the dynamic effects are included. The improvements over the prior model are that it includes the dynamic stall effect, pitching circulation and added mass effect. They repeated the experiment with the test model as is mentioned in Ref. [30] and found appreciable variations with the prior results. The correlation with their calculated values by the dynamic Vortex model and the latest experimental results of the local blade forces and wake velocities seem to be reasonable in some cases.

In 1984, Cardona [31] incorporated the effect of flow curvature following the method given by Migliore et al. [32] into the original aerodynamic Vortex model presented by Strickland et al. [30]. They also added a modified form of the dynamic stall effects. They found an improved correlation with the calculated and experimental results for the instantaneous aerodynamic blade forces and the overall power coefficients.

The main disadvantage of Vortex model is that it takes too much computation time. Furthermore, this model still rely on significant simplifications, like potential flow is assumed in the wake and the effect of viscosity in the blade aerodynamics is included through empirical force coefficients [33].

5.3. Cascade model

The periodic equidistant arrangement of several blades or vanes of turbomachinaries is called a cascade. Hence, the cascade is the basic element of the turbomachine, and cascade flow is the essential physical phenomenon for the operation of the machine [34]. The Cascade model was proposed by Hirsch and Mandal [35] to apply the cascade principles, widely used for turbomachinaries, for the analysis of VAWTs for the first time. In this model, the blade airfoils of a turbine are assumed to be positioned in a plane surface (termed as the cascade) with the blade interspace equal to the turbine circumferential distance divided by the number of blades as shown in Fig. 12. The relationship between the

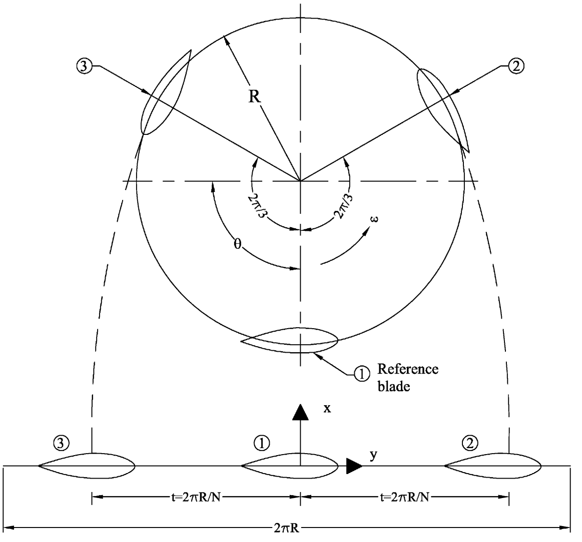


Fig. 12. Development of blade into a cascade configuration.

wake velocity and the free stream velocity is established by using Bernoulli's equation while the induced velocity is related to the wake velocity through a particular semi-empirical expression.

In this model, the aerodynamic characteristics of each element of the blade are obtained independently, like the double-multiple streamtube theory, for the upwind and downwind halves of the rotor considering the local Reynolds number and the local angle of attack as shown in Fig. 13. After determination of the local relative flow velocity and the angle of attack, the VAWT is developed into a cascade configuration that is shown in Fig. 12. The cascade is considered in a plane normal to the turbine axis. If the blade represented by ① at an azimuth angle θ is considered as the reference blade, the flow conditions on the other two blades represented by ② and ③, are assumed to be equal to those of the reference blade. This process is continued for one complete revolution of the reference blade with a step of $\delta\theta$.

To find the induced velocity, a relationship between the wake velocity and the induced velocity is introduced. For the upstream side this is expressed as

$$\frac{V_{\mathrm{au}}}{V_\infty} = \left(\frac{V_{\mathrm{e}}}{V_\infty}\right)^{k_{\mathrm{i}}} \tag{26}$$

and for the downstream side, this is expressed as

$$\frac{V_{\mathrm{ad}}}{V_{\mathrm{e}}} = \left(\frac{V_{\mathrm{w}}}{V_{\mathrm{e}}}\right)^{k_{\mathrm{i}}}, \tag{27}$$

where V_{e} and V_{w} are the wake velocities in the upstream and downstream side. The value of the exponent k_{i} is found from a fit of experimental results. The induced velocity ratio for

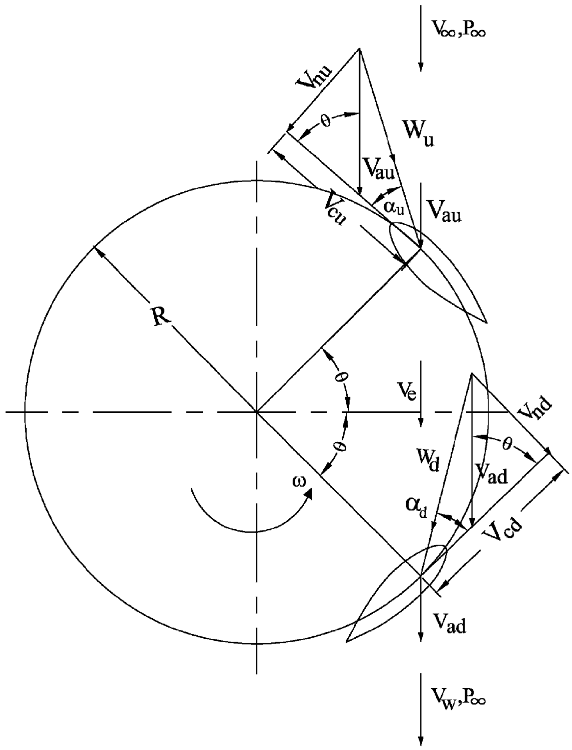


Fig. 13. Horizontal section of a straight-bladed Darrieus-type VAWT with flow velocities in the upstream and downstream sides.

the downstream side can be written in the form,

$$\frac{V_{\mathrm{ad}}}{V_\infty}=\frac{V_{\mathrm{ad}}}{V_{\mathrm{e}}}\frac{V_{\mathrm{e}}}{V_\infty}=\frac{V_{\mathrm{e}}}{V_\infty}\left(\frac{V_{\mathrm{w}}}{V_{\mathrm{e}}}\right)^{k_{\mathrm{i}}}. \tag{28}$$

The expression of the exponent k_{i} is written in accordance with Ref. [35] as

$$k_{\mathrm{i}}=(0.425+0.332\sigma), \tag{29}$$

where $\sigma = NC/R$. The final expression for the overall torque is found from,

$$Q=\rho R^3\frac{H}{R}\int_0^{2\pi}(W_2^2\sin\alpha_2\cos\alpha_2-W_1^2\sin\alpha_1\cos\alpha_1)\,\mathrm{d}\theta, \tag{30}$$

where W_1 and W_2 are the relative velocities in the cascade inlet and outlet. Detail description of this model can be found from Hirsh and Mandal [35].

The Cascade model can predict the overall values of both low and high solidity turbines quite well. It takes reasonable computation time. It does not make any convergence problem even at the high tip speed ratios and high solidities. The instantaneous blade forces calculated by this model show improved correlation in comparison to those calculated by the conventional Momentum model. The theory also incorporated the effect of the local Reynolds number variation at different azimuth angles (orbital position),

zero-lift-drag coefficients, finite aspect ratios and the flow curvature effect in the calculation process.

Subsequently, to improve the analytical capability of this model, two important effects of the dynamic stall and flow curvature with blade pitching were taken into account by Mandal and Burton [36]. The calculated values of the wake velocities after these modifications become comparable with those by the complex dynamic Vortex model.

6. Conclusions

Several aerodynamic models have been analyzed in this paper which are applied for better performance prediction and design analysis of straight-bladed Darrieus-type VAWT. At present the most widely used models are the double-multiple streamtube model, free-Vortex model and the Cascade model. It has been found that, each of these three models has their strengths and weaknesses. Though among these three models, the Vortex models are considered to be the most accurate models according to several researchers, but they are computationally very expensive and in some cases they suffer from convergence problem. It has also been found that the double-multiple streamtube model is not suitable for high tip speed ratios and high-solidity VAWT. On the other hand, the Cascade model gives smooth convergence even in high tip speed ratios and high solidity VAWT with quite reasonable accuracy.

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